High-energy behavior of hadronic total cross sections from lattice QCD

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Abstract

By means of a nonperturbative approach to *soft* high-energy hadron-hadron scattering, based on the analytic continuation of Wilson-loop correlation functions from Euclidean to Minkowskian theory, we shall investigate the asymptotic energy dependence of hadron-hadron total cross sections in lattice QCD: we will show, using best fits of the lattice data with proper functional forms satisfying unitarity and other physical constraints, how indications emerge in favor of a universal asymptotic high-energy behavior of the kind $B\log^2 s$ for hadronic total cross sections.

Keywords:

1. Introduction

Among the oldest open problems of hadronic physics, not yet satisfactorily solved in QCD, there is the problem of predicting hadronic total cross sections at high energy from first principles. Present-day experimental observations (up to a center-of-mass total energy $\sqrt{s} = 7$ TeV, reached at the LHC pp collider [1]) seem to support the following asymptotic high-energy behavior: $\sigma_{\rm tot}^{(hh)}(s) \sim B\log^2 s$, with a *universal* (i.e., *not* depending on the particular hadrons involved) coefficient $B \simeq 0.3$ mb [2]. This behavior is consistent with the well-known *Froissart-Lukaszuk-Martin* (FLM) *theorem* [3], according to which, for $s \to \infty$, $\sigma_{\rm tot}^{(hh)}(s) \le (\pi/m_\pi^2)\log^2(s/s_0)$, where m_π is the pion mass and s_0 is an unspecified squared mass scale. As we believe QCD to be the fundamental theory of strong interactions, we also expect that it correctly predicts from first principles the behavior of hadronic total cross sections. However, in spite of all the efforts, a satisfactory solution to this problem is still lacking. (For some theoretical supports to the universality of B, see Ref. [4] and references therein.)

This problem is part of the more general problem of high-energy elastic scattering at low transferred momentum, the so-called *soft high-energy scattering*. As soft high-energy processes possess two different energy scales, the total center-of-mass energy squared s and the transferred momentum squared t, smaller than the typical energy scale of strong interactions ($|t| \lesssim 1$ GeV^2 $\ll s$), we cannot fully rely on perturbation theory (PT). A nonperturbative (NP) approach in the framework of QCD has been proposed in [5] and further developed in [6]: using a functional integral approach, high-energy hadron-hadron elastic scattering amplitudes are shown to be governed by the correlation function (CF) of certain Wilson loops defined in Minkowski space [6]. This CF can be reconstructed by *analytic continuation* from the CF of two Euclidean Wilson loops [7–9], that can be calculated using the NP methods of Euclidean Field Theory. The analytic-continuation relations have allowed the NP investigation of CFs using some analytical models, such as the *Stochastic Vacuum Model* (SVM) [10], the *Instanton Liquid Model* (ILM) [11, 12], the AdS/CFT *correspondence* [13], and they have also allowed a numerical study by Monte Carlo simulations in *Lattice Gauge Theory* (LGT) [12, 14].

In what follows, after a brief survey of the NP approach to soft high-energy scattering in the case of meson-meson *elastic* scattering, and of the numerical approach based on LGT, we will focus on the search for a new parameterization of the (Euclidean) CF that, in order: *i*) fits well the lattice data; *ii*) satisfies unitarity after analytic continuation; and, most importantly, *iii*) leads to a rising behavior of total cross sections at high energy as $B\log^2 s$, in agreement with experimental data [15]. In our approach, the coefficient B turns out to be *universal*, i.e, the same for all hadronic scattering pro-

[*]Speaker at the conference.

[**]Supported by MICINN [CPAN project CSD2007-00042 (Consolider-Ingenio2010 program); grant FPA2009-09638].

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cesses, being related to the mass-scale μ which sets the large impact-parameter behavior of the CF.

2. High-energy meson-meson elastic scattering amplitude and Wilson-loop correlation functions

In the *soft* high-energy regime, the elastic scattering amplitude $\mathcal{M}_{(hh)}$ of two *mesons*, of the same mass m for simplicity, can be reconstructed from the scattering amplitude $\mathcal{M}_{(dd)}$ of two dipoles of fixed transverse sizes $\vec{r}_{1,2\perp}$, and fixed longitudinal-momentum fractions $f_{1,2}$ of the quarks in the two dipoles, after folding with squared wave functions $\rho_{1,2} = |\psi_{1,2}|^2$ describing the interacting hadrons [6],

$$\mathcal{M}_{(hh)}(s,t) = \int d^2\nu\, \rho_1(\nu_1)\rho_2(\nu_2)\mathcal{M}_{(dd)}(s,t;\nu_1,\nu_2) \equiv \langle\langle \mathcal{M}_{(dd)}(s,t;1,2)\rangle\rangle, \quad (1)$$

where $\nu_i = (\vec{r}_{i\perp}, f_i)$ denotes collectively the dipole variables, $d^2\nu = d\nu_1 d\nu_2$, $\int d\nu_i = \int d^2\vec{r}_{i\perp}\int_0^1 df_i$, and $\int d\nu_i\ \rho_i(\nu_i) = 1$. In turn, the dipole-dipole (dd) scattering amplitude is obtained from the (properly normalized) CF of two Wilson loops (WL) in the fundamental representation, defined in Minkowski spacetime, running along the paths made up of the quark and antiquark classical straight-line trajectories, and thus forming a hyperbolic angle $\chi \simeq \log(s/m^2)$ in the longitudinal plane. The paths are cut at proper times $\pm T$ as an infrared regularization, and closed by straight-line "links" in the transverse plane, in order to ensure gauge invariance; eventually, $T \to \infty$. It has been shown in [7–9] that the relevant Minkowskian CF $\mathcal{G}_M(\chi; T; \vec{z}_\perp; \nu_1, \nu_2)$ ($\vec{z}_\perp$ being the *impact parameter*, i.e., the transverse separation between the two dipoles) can be reconstructed, by means of *analytic continuation*, from the Euclidean CF of two Euclidean WL, $\mathcal{G}_E(\theta; T; \vec{z}_\perp; \nu_1, \nu_2) \equiv \langle \mathcal{W}_1^{(T)}\mathcal{W}_2^{(T)}\rangle/(\langle\mathcal{W}_1^{(T)}\rangle\langle\mathcal{W}_2^{(T)}\rangle) - 1$, where $\langle\ldots\rangle$ is the average in the sense of the Euclidean QCD functional integral. The Euclidean WL $\mathcal{W}_{1,2}^{(T)} = N_c^{-1}\mathrm{Tr}\{T\exp[-ig\oint_{\mathcal{C}_{1,2}} A_\mu(x)dx_\mu]\}$ are calculated on the following quark [q]-antiquark [$\overline{q}$] straight-line paths, $\mathcal{C}_i : X_i^{q[\overline{q}]}(\tau) = z_i + \frac{p_i}{m}\tau + f_i^{q[\overline{q}]} r_i$, with $\tau \in [-T, T]$, and closed by straight-line paths in the transverse plane at $\tau = \pm T$. Here $p_{1,2} = m(\pm\sin\frac{\theta}{2}, \vec{0}_\perp, \cos\frac{\theta}{2})$, $r_i = (0, \vec{r}_{i\perp}, 0)$, $z_i = \delta_{i1}(0, \vec{z}_\perp, 0)$ and $f_i^q \equiv 1 - f_i$, $f_i^{\overline{q}} \equiv -f_i$. We define also the CFs with the infrared cutoff removed as $\mathcal{C}_{E,M} \equiv \lim_{T\to\infty}\mathcal{G}_{E,M}$. The dd scattering amplitude is then obtained from $\mathcal{C}_E(\theta;\ldots)$ [with $\theta \in (0,\pi)$] by means of analytic continuation as ($t = -|\vec{q}_\perp|^2$)

$$\mathcal{M}_{(dd)}(s,t;\nu_1,\nu_2) \equiv -i\,2s\int d^2\vec{z}_\perp e^{i\vec{q}_\perp\cdot\vec{z}_\perp}\mathcal{C}_M(\chi;\vec{z}_\perp;\nu_1,\nu_2) = -i\,2s\int d^2\vec{z}_\perp e^{i\vec{q}_\perp\cdot\vec{z}_\perp}\mathcal{C}_E(\theta\to -i\chi;\vec{z}_\perp;\nu_1,\nu_2)\,. \quad (2)$$

Choosing $\rho_{1,2}$ invariant under rotations and under the exchange $f_i \to 1 - f_i$ (see Refs. [6]), $\mathcal{C}_E$ can be substituted in (1) with the following *averaged* CF: $\mathcal{C}_E^{ave}(\theta; |\vec{z}_\perp|; \hat{\nu}_1, \hat{\nu}_2) \equiv \int\frac{d\phi_1}{2\pi}\int\frac{d\phi_2}{2\pi}\frac{1}{4}\{\mathcal{C}_E(\theta;\vec{z}_\perp;\nu_1,\nu_2) + \mathcal{C}_E(\theta;\vec{z}_\perp;\overline{\nu}_1,\nu_2) + \mathcal{C}_E(\theta;\vec{z}_\perp;\nu_1,\overline{\nu}_2) + \mathcal{C}_E(\theta;\vec{z}_\perp;\overline{\nu}_1,\overline{\nu}_2)\}$, where $\vec{r}_{i\perp} = |\vec{r}_{i\perp}|(\cos\phi_i, \sin\phi_i)$, $\hat{\nu}_i = (|\vec{r}_{i\perp}|, f_i)$ and $\overline{\nu}_i = (-\vec{r}_{i\perp}, 1 - f_i)$. Similarly, one defines the Minkowskian averaged CF, $\mathcal{C}_M^{ave}$. As a consequence of the (Euclidean) *crossing-symmetry relations* [16], $\mathcal{C}_E(\pi-\theta;\vec{z}_\perp;\nu_1,\nu_2)$= $\mathcal{C}_E(\theta;\vec{z}_\perp;\nu_1,\overline{\nu}_2)$=$\mathcal{C}_E(\theta;\vec{z}_\perp;\overline{\nu}_1,\nu_2)$, $\mathcal{C}_E^{ave}$ is automatically *crossing-symmetric*, i.e., $\mathcal{C}_E^{ave}(\pi-\theta;\ldots) = \mathcal{C}_E^{ave}(\theta;\ldots)$.

3. Wilson-loop correlation functions on the lattice and comparison with known analytical results

In Refs. [12, 14] two of us performed a Monte Carlo calculation of $\mathcal{C}_E$ in *quenched* QCD at lattice spacing $a(\beta = 6) \simeq 0.1$ fm, on a 16^4 hypercubic lattice. We used loops of transverse size a at angles $\cot\theta = 0, \pm 1, \pm 2$. The longitudinal-momentum fractions were set to $f_{1,2} = \frac{1}{2}$ without loss of generality [12]. We studied the configurations $\vec{z}_\perp \parallel \vec{r}_{1\perp} \parallel \vec{r}_{2\perp}$ ("zzz"), $\vec{z}_\perp \perp \vec{r}_{1\perp} \parallel \vec{r}_{2\perp}$ ("*zyy*") in the transverse plane, and the averaged quantity ("*ave*") defined above, for loops at transverse distances $d \equiv |\vec{z}_\perp|/a = 0, 1, 2$.

Numerical simulations of LGT provide (within the errors) the true QCD expectation for $\mathcal{C}_E$; approximate analytical calculations of $\mathcal{C}_E$ have then to be compared with the lattice data, in order to test the goodness of the approximations involved. $\mathcal{C}_E$ has been evaluated in the SVM, $\mathcal{C}_E^{(\mathrm{SVM})} = \frac{2}{3}e^{-\frac{1}{3}K_S\cot\theta} + \frac{1}{3}e^{\frac{2}{3}K_S\cot\theta} - 1$ [10], in PT, $\mathcal{C}_E^{(\mathrm{PT})} = K_p\cot^2\theta$ [8, 10, 17], in the ILM, $\mathcal{C}_E^{(\mathrm{ILM})} = \frac{K_I}{\sin\theta}$ [11, 12], and, using the AdS/CFT correspondence, for planar, strongly coupled $\mathcal{N} = 4$ SYM at large $|\vec{z}_\perp|$, $\mathcal{C}_E^{(\mathrm{AdS/CFT})} = e^{\frac{K_1}{\sin\theta} + K_2\cot\theta + K_3\cos\theta\cot\theta} - 1$ [13]. The coefficients $K_i = K_i(\vec{z}_\perp;\nu_1,\nu_2)$ are functions of $\vec{z}_\perp$ and of the dipole variables $\vec{r}_{i\perp}, f_i$. The comparison of the lattice data with these analytical calculations, performed in Ref. [14] by fitting the lattice data with the corresponding functional form, is not fully satisfactory, even though largely improved best fits have been obtained by combining the ILM and PT expressions into the expression $\mathcal{C}_E^{(\mathrm{ILMp})} = \frac{K_{\mathrm{Ip1}}}{\sin\theta} + K_{\mathrm{Ip2}}\cot^2\theta$. Regarding the energy dependence of total cross sections, the above analytical models are absolutely unsatisfactory, as they do not lead to *Froissart-like* total cross sections at high energy, as experimental data seem to suggest. Infact, the SVM, PT, ILM and ILMp parameterizations lead to asymptotically constant $\sigma_{\mathrm{tot}}^{(hh)}$, while the AdS/CFT result leads to power-like $\sigma_{\mathrm{tot}}^{(hh)}$ [18].

4. How a Froissart-like total cross section can be obtained

We will now introduce, and partially justify, new parameterizations of the CF that: i) fit well the data; ii) satisfy the unitarity condition after analytic continuation; and iii) lead to total cross sections rising as $B \log^2 s$ in the high-energy limit [15]. Regarding unitarity, from (1) and (2) one recognizes that the quantity $A(s, |\vec{z}_\perp|) \equiv \langle\langle C_M(\chi; \vec{z}_\perp; 1, 2)\rangle\rangle$ is the scattering amplitude in impact-parameter space, which must satisfy the *unitarity constraint* $|A + 1| \leq 1$ (see [19]). Since $\int d\nu_i\, \rho_i(\nu_i) = 1$, this is the case if the following *sufficient* condition is satisfied (as we can replace $C_M \to C_M^{ave}$ when averaging over the dipole variables, a similar but weaker *sufficient* condition can be given in terms of C_M^{ave}):

$$|C_M(\chi; \vec{z}_\perp; \nu_1, \nu_2) + 1| \leq 1 \qquad \forall\ \vec{z}_\perp,\ \nu_1,\ \nu_2. \quad (3)$$

The conditions above constrain rather strongly the possible parameterizations. For example, conditions ii) and iii) cannot be simultaneously satisfied if the angular dependence can be factorized, for in this case the unitarity constraint would imply $\sigma_{\rm tot}^{(hh)}(\chi) \to$ const. for $\chi \to \infty$. We shall *assume* that the Euclidean CF can be written as $C_E = \exp K_E - 1$, where $K_E = K_E(\theta; \vec{z}_\perp; \nu_1, \nu_2)$ is a *real* function (since C_E is *real* [14]). This assumption is rather well justified: in the large-N_c expansion, $C_E \sim O(1/N_c^2)$, so that $C_E + 1 \geq 0$ is certainly satisfied for large N_c; all the known analytical models satisfy it; the lattice data of Refs. [12, 14] confirm it. The Minkowskian CF is then obtained after analytic continuation: $C_M = \exp K_M - 1$, with $K_M(\chi; \ldots) = K_E(\theta \to -i\chi; \ldots)$. At large χ, C_M is expected to obey the unitarity condition (3), which in this case reduces to ${\rm Re} K_M \leq 0\ \ \forall \vec{z}_\perp, \nu_1, \nu_2$.

For a *confining* theory like QCD, C_E is expected to decay exponentially as $C_E \sim \alpha\, e^{-\mu|\vec{z}_\perp|}$ at large $|\vec{z}_\perp|$, where μ is some mass-scale proportional to the mass of the lightest glueball ($M_G \simeq 1.5$ GeV) or maybe to the inverse of the so-called *vacuum correlation length* λ_{vac} (e.g., $\mu = 2/\lambda_{vac}$ in the SVM), which has been measured on the lattice [20], both in *quenched* ($\lambda_{vac} \simeq 0.22$ fm) and *full* QCD ($\lambda_{vac} \simeq 0.30$ fm). Therefore, we should require the same large-$|\vec{z}_\perp|$ behavior for K_E, i.e., $K_E \sim e^{-\mu|\vec{z}_\perp|}$.

Let us now assume that the leading term of the Minkowskian CF for $\chi \to +\infty$ is of the form $C_M \sim \exp\left(i\beta\, f(\chi)\, e^{-\mu|\vec{z}_\perp|}\right) - 1$ [recall $\chi \simeq \log(s/m^2)$], where $\beta = \beta(\nu_1, \nu_2)$ is a function of the dipole variables and $f(\chi)$ is a *real* function such that $f(\chi) \to +\infty$ for $\chi \to +\infty$. In this case, the unitarity condition (3) is equivalent (for large χ) to ${\rm Im}\beta \geq 0$. This $\vec{z}_\perp$ dependence is expected to be valid only for large enough $|\vec{z}_\perp|$, but for simplicity we shall first assume that it is valid $\forall |\vec{z}_\perp| \geq 0$. By virtue of the *optical theorem*, $\sigma_{\rm tot}^{(hh)}(s) \sim s^{-1} {\rm Im}\mathcal{M}_{(hh)}(s, t = 0)$, we have that $\sigma_{\rm tot}^{(hh)} \sim 4\pi\mu^{-2} {\rm Re}\langle\langle J(\eta, \beta)\rangle\rangle$, where $J(\eta, \beta) \equiv \int_0^\infty dy\, y[1 - \exp(i\beta e^{\eta - y})]$, with $f(\chi) = e^\eta$, and $y = \mu|\vec{z}_\perp|$. Expanding the exponential, integrating term by term, and deriving with respect to η, we find $\partial J/\partial\eta = -\sum_{n=1}^\infty (-z)^n/(n!n) = E_1(z) + \log(z) + \gamma$, for $|\arg(z)| < \pi$ ($z = -i\beta e^\eta$), where γ is the Euler-Mascheroni constant and $E_1(z)$ is Schlömilch's exponential integral (see [21]). Since $E_1(z) \sim e^{-z}/z$ at large $|z|$, for ${\rm Re}\, z \geq 0 \Leftrightarrow {\rm Im}\beta \geq 0$, the asymptotic form of $\partial J/\partial\eta$ is readily obtained; re-integrating in η and substituting back $\eta = \log f(\chi)$, we find $\sigma_{\rm tot}^{(hh)} \sim 4\pi\mu^{-2}\langle\langle \frac{1}{2}\log^2 f(\chi) + \log f(\chi)(\log|\beta| + \gamma) + \ldots\rangle\rangle$. If one takes $f(\chi) = \chi^p e^{n\chi}$, the resulting asymptotic behavior of $\sigma_{\rm tot}^{(hh)}$ is

$$\sigma_{\rm tot}^{(hh)} \sim B \log^2 s, \qquad \text{with:} \qquad B = \frac{2\pi n^2}{\mu^2}. \quad (4)$$

The same result is obtained assuming the above approximation for C_M only for $|\vec{z}_\perp| > z_0 \gg \mu^{-1}, |\vec{r}_{i\perp}|$: the difference in $\sigma_{\rm tot}^{(hh)}$, coming from the integration of C_M over the finite region $|\vec{z}_\perp| < z_0$, is bounded by a constant due to the unitarity constraint. The analysis can be repeated for C^{ave} without altering any conclusion. We want to emphasize that the above result is *universal*, depending only on the mass scale μ, which sets the large-$|\vec{z}_\perp|$ dependence of the CF, since the integration over the dipole variables does *not* affect the leading term. The *universal* coefficient B is not affected by the masses of the scattering particles: for mesons of masses $m_{1,2}$, the rapidity becomes $\chi \sim \log(\frac{s}{m_1 m_2})$, which simply corresponds to a change of the energy scale implicitly contained in (4).

5. New analysis of the lattice data

We show now three parameterizations $C_E^{(i)} = \exp K_E^{(i)} - 1$, $i = 1, 2, 3$, that satisfy the criteria i)–iii) listed above, together with the corresponding estimate of the asymptotic total cross section at high energy [15]. We focus our analysis on the averaged CF C^{ave}, that is "closer" to the hadronic scattering matrix $\mathcal{M}_{(hh)}$. As C^{ave} is *crossing-symmetric*, so are our parameterizations.

In order to parameterize K_E, a possible strategy is to combine known QCD results and variations thereof. We have then exponentiated the two-gluon exchange and the one-instanton contribution (i.e., the ILMp expression), adding a term which could yield a rising cross section, e.g., a term proportional to $\cos\theta\cot\theta$, as in the AdS/CFT result. We thus find the following parameterization: $K_E^{(1)} = \frac{K_1}{\sin\theta} + K_2\cot^2\theta + K_3\cos\theta\cot\theta$. Another

$\chi^2_{\text{d.o.f.}}$	$d = 0$	$d = 1$	$d = 2$
Corr 1	2.81	1.25	0.05
Corr 2	0.55	0.31	0.05
Corr 3	0.17	0.11	0.10

Table 1: Chi-squared per degree of freedom for a best fit with the indicated function.

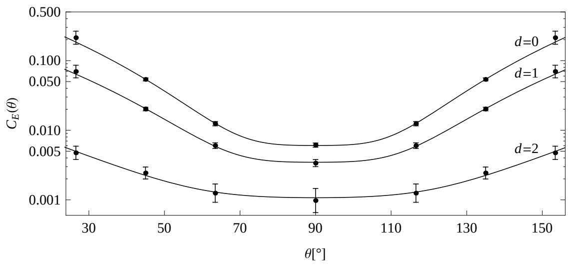

Figure 1: Lattice data for C_E^{ave} and best fit with $C_E^{(3)}$.

strategy is suggested again by the AdS/CFT result: one can try to adapt to QCD analytical expressions obtained in related models, such as $\mathcal{N}=4$ SYM. Although QCD, of course, is not $\mathcal{N} = 4$ SYM, it is sensible to assume a similar dependence on θ (basically assuming the existence of the yet unknown gravity dual for QCD). In this spirit, the second parameterization that we propose is: $K_E^{(2)} = \frac{K_1}{\sin\theta} + K_2(\frac{\pi}{2} - \theta)\cot\theta + K_3\cos\theta\cot\theta$. Beside the AdS/CFT-like terms, it contains also a $\theta\cot\theta$ term. Our last parameterization is: $K_E^{(3)} = \frac{K_1}{\sin\theta} + K_2(\frac{\pi}{2} - \theta)^3\cos\theta$. While the first term is "familiar", the second one is not present in the known analytical models, but it is a fact that the resulting best fit is extremely good (see Fig. 1). In Table 1 we report the values of the chi-squared per degree of freedom ($\chi^2_{\text{d.o.f.}}$) of the best fits to the lattice data.

In the three cases, the unitarity condition $\mathrm{Re}K_M^{(i)} \leq 0$ is satisfied if $K_2 \geq 0$: this is actually the case for our best fits (within the errors). The leading term after analytic continuation is of the form $\chi^p e^\chi$ which, according to (4), leads to $\sigma_{\text{tot}}^{(hh)} \sim B\log^2 s$. The value of $B = 2\pi/\mu^2$, obtained through a fit of the coefficient of the leading term with an exponential function $\sim e^{-\mu|\vec{z}_\perp|}$ over the available distances, is found to be compatible with the experimental result (within the large errors) in all the three cases (see Table 2). However, this must be taken only as an estimate, as lattice data are available only for small $|\vec{z}_\perp|$.

6. Conclusions

We have shown how a *universal* and *Froissart-like* hadron-hadron total cross section at high energy can emerge in QCD, and we have found indications for

	μ (GeV)	$\lambda = \frac{1}{\mu}$ (fm)	$B = \frac{2\pi}{\mu^2}$ (mb)
Corr 1	4.64(2.38)	$0.042^{+0.045}_{-0.014}$	$0.113^{+0.364}_{-0.037}$
Corr 2	3.79(1.46)	$0.052^{+0.032}_{-0.014}$	$0.170^{+0.277}_{-0.081}$
Corr 3	3.18(98)	$0.062^{+0.028}_{-0.015}$	$0.245^{+0.263}_{-0.100}$

Table 2: Mass-scale μ, "decay length" $\lambda = 1/\mu$ and the coefficient $B = 2\pi/\mu^2$ obtained with our parameterizations.

this behavior from the lattice. The functional integral approach provides the "natural" setting for achieving this result, since it encodes the energy dependence of hadronic scattering amplitudes in a single *elementary* object, i.e., the loop-loop CF.

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